EE105 – Fall 2014
Microelectronic Devices and Circuits

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Amplifier Transfer Functions

$$A_v(s) = \frac{V_o(s)}{V_s(s)} \qquad s = \sigma + j\omega$$

$A_v(s)$ = Frequency-dependent voltage gain
$V_o(s)$ and $V_s(s)$ = Laplace Transforms of input and output voltages of amplifier,

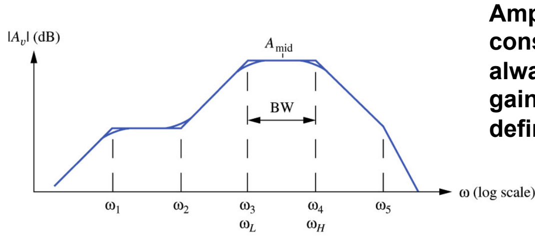
$$A_v(s) = K\frac{(s+z_1)(s+z_2)..(s+z_m)}{(s+p_1)(s+p_2)..(s+p_3)}$$ **(In factorized form)**

$(-z_1, -z_2, \ldots -z_m)$ = zeros (frequencies for which transfer function is zero)
$(-p_1, -p_2, \ldots -p_m)$ = *poles* (frequencies for which transfer function is infinite)

$$A_v(j\omega) = |A_v(j\omega)|\angle A_v(j\omega)$$ **(In polar form)**

Bode plots display magnitude of the transfer function in dB and the phase in degrees (or radians) on a logarithmic frequency scale.

Complex Transfer Functions



$$A_v(s) = \frac{Ks(s+\omega_2)}{(s+\omega_1)(s+\omega_3)(s+\omega_4)(s+\omega_5)}$$

$$A_v(s) = \frac{A_{\text{mid}}s(s+\omega_2)}{(s+\omega_1)(s+\omega_3)\left(\frac{s}{\omega_4}+1\right)\left(\frac{s}{\omega_5}+1\right)}$$

Amplifier has 2 frequency ranges with constant gain. The mid-band region is always defined as region of highest gain and cutoff frequencies are defined in terms of midband gain.

$$|A_v(j\omega_L)| = |A_v(j\omega_H)| = \frac{A_{\text{mid}}}{\sqrt{2}}$$

For widely spaced poles as in the figure,

$\omega_H \cong \omega_4$ and $\omega_L \cong \omega_3$,

$$\text{BW} = f_4 - f_3 = \frac{\omega_4 - \omega_3}{2\pi}$$

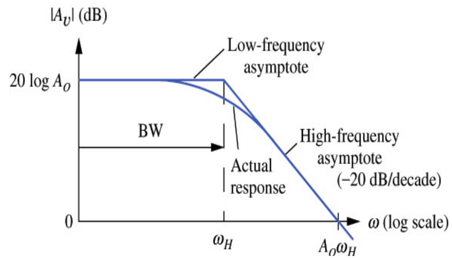
Low-Pass Amplifiers

- **Amplifies signals below a cut-off frequency, including dc.**
- **Most operational amplifiers are designed as low pass amplifiers.**
- **Simplest (single-pole) low-pass amplifier is described by**

$$A_v(s) = A_o \frac{\omega_H}{s+\omega_H} = \frac{A_o}{1+\frac{s}{\omega_H}}$$

  - **A_o = low-frequency gain or mid-band gain**
  - **ω_H = upper cutoff frequency or upper half-power point of amplifier.**

Low-pass Amplifier Magnitude Response



$$\left|A_v(j\omega)\right| = \left|\frac{A_o\omega_H}{j\omega+\omega_H}\right| = \frac{\left|A_o\omega_H\right|}{\sqrt{\omega^2+\omega_H^2}} = \frac{\left|A_o\right|}{\sqrt{\frac{\omega^2}{\omega_H^2}+1}}$$

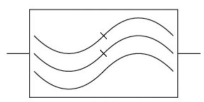
For $\omega \ll \omega_H$, $\left|A_v(j\omega)\right|_{dB} = A_o$

For $\omega \gg \omega_H$, $\left|A_v(j\omega)\right| = \dfrac{A_o\omega_H}{\omega}$

For $\omega = \omega_H$, $\left|A_v(j\omega)\right| = \dfrac{A_o}{\sqrt{2}}$

$\Rightarrow \omega_H$: 3-dB Bandwidth in Rad/s

$\Rightarrow f_H = \dfrac{\omega_H}{2\pi}$: 3-dB Bandwidth in Hz

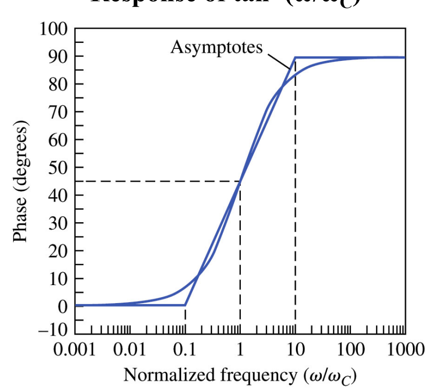


Low-pass filter symbol

- **Gain is unity (0 dB) at $\omega = A_o\omega_H = \omega_T$ called gain-bandwidth product**
- **Bandwidth (frequency range with constant amplification) = ω_H (rad/s)**

Arc Tangent Phase Response

Response of $\tan^{-1}(\omega/\omega_C)$



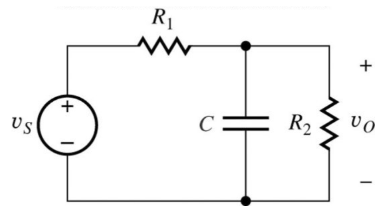
If A_o positive: phase angle = 0^0 at dc
(If A_o negative: phase angle = 180^0)

At ω_C: phase = 45^0
One decade below ω_C: phase = 5.7^0
One decade above ω_C: phase = 84.3^0
Two decades below ω_C: phase = 0^0
Two decades above ω_C: phase = 90^0

LPF Response:

$$\angle A_v(j\omega) = \angle\frac{A_o}{1+j\dfrac{\omega}{\omega_H}} = \angle A_o - \tan^{-1}\left(\frac{\omega}{\omega_H}\right)$$

RC Low-pass Filter



$$V_o = V_s \frac{\dfrac{\dfrac{R_2}{sC}}{R_2 + \dfrac{1}{sC}}}{R_1 + \dfrac{\dfrac{R_2}{sC}}{R_2 + \dfrac{1}{sC}}}$$

$$\text{Impedance of capacitor} = \frac{1}{sC}$$

$$\omega_H = \frac{1}{(R_1 \| R_2)C}$$

$$\frac{V_o}{V_s} = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{1}{1 + \dfrac{s}{\omega_H}}\right)$$

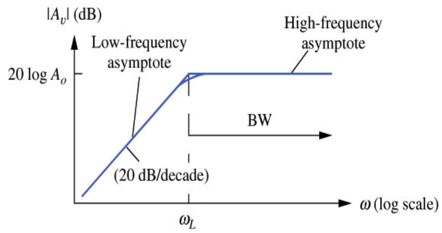
High-pass Amplifiers

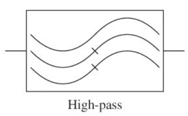
- **Combines a single pole with a zero at the origin.**
- **Simplest high-pass amplifier is described by**

$$A_v(s) = \frac{A_o s}{s + \omega_L}$$

- **ω_L = lower cutoff frequency or lower half-power point of amplifier.**

High-pass Amplifier
Magnitude and Phase Response





High-pass filter symbol

$$|A_v(j\omega)| = \left|\frac{A_O j\omega}{j\omega+\omega_L}\right| = \frac{|A_o|\omega}{\sqrt{\omega^2+\omega_L^2}} = \frac{|A_o|}{\sqrt{1+\frac{\omega_L^2}{\omega^2}}}$$

For $\omega >> \omega_L$, $|A_v(j\omega)| = A_o$

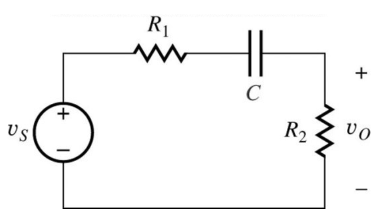
For $\omega << \omega_L$, $|A_v(j\omega)| = \dfrac{A_o\omega}{\omega_L}$

For $\omega = \omega_L$, $|A_v(j\omega)| = \dfrac{A_o}{\sqrt{2}}$

- **Bandwidth (frequency range with constant amplification) is infinite**
- **Phase response is given by**

$$\angle A_v(j\omega) = \angle \frac{A_o j\omega}{j\omega + j\omega_L} = \angle A_o + 90^0 - \tan^{-1}\left(\frac{\omega}{\omega_L}\right)$$

RC High-pass Filter



$$V_o = V_s \frac{R_2}{R_1 + \frac{1}{sC} + R_2}$$

$$\therefore \quad \frac{V_o}{V_s} = \left(\frac{R_2}{R_1+R_2}\right)\left(\frac{s}{s+\omega_L}\right)$$

$$\omega_L = \frac{1}{(R_1+R_2)C}$$

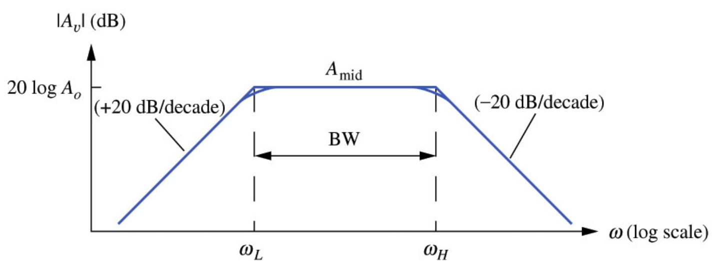
Band-pass Amplifiers

- A band-pass characteristic is obtained by combining high-pass and low-pass characteristics.
- Transfer function of a band-pass amplifier is given by

$$A_v(s) = \frac{A_o s\omega_H}{(s+\omega_L)(s+\omega_H)} = A_o \frac{s}{(s+\omega_L)} \frac{1}{\left(\frac{s}{\omega_H}+1\right)}$$

- An ac-coupled amplifier has a band-pass characteristic:
  - Capacitors added to circuit cause low frequency roll-off
  - Inherent frequency limitations of solid-state devices cause high-frequency roll-off.

Band-pass Amplifier Magnitude and Phase Response



- The mid-band range of frequencies is given by

$$\omega_L \le \omega \le \omega_H \qquad |A_v(j\omega)| \cong A_o$$

- Transfer characteristic is

$$|A_v(j\omega)| = \left|\frac{A_o(j\omega)\omega_H}{(j\omega+\omega_L)(j\omega+\omega_H)}\right| = \frac{A_o\omega\omega_H}{\sqrt{(\omega^2+\omega_L^2)(\omega^2+\omega_H^2)}}$$

Band-pass Amplifier
Magnitude and Phase Response (cont.)

At both ω_H and ω_L, assuming $\omega_L << \omega_H$

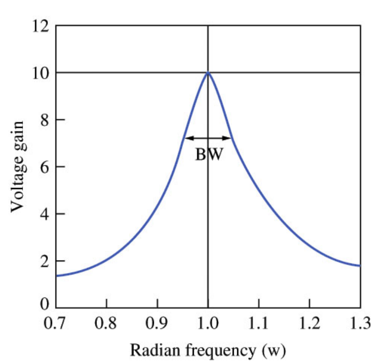
$$\left|A_v(j\omega_L)\right| = \left|A_v(j\omega_H)\right| = \frac{A_o}{\sqrt{2}}$$

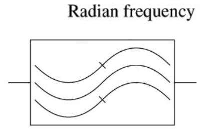
Bandwidth = ω_H - ω_L

The phase response is given by

$$\angle A_v(j\omega) = \angle \frac{A_o j\omega}{j\omega + j\omega_L} = \angle A_o + 90^0 - \tan^{-1}\left(\frac{\omega}{\omega_L}\right) - \tan^{-1}\left(\frac{\omega}{\omega_H}\right)$$

Narrow-band or High-Q Band-pass Amplifiers





Band-pass filter symbol

- Gain maximum at center frequency ω_o and decreases rapidly by 3 dB at ω_H and ω_L.
- Bandwidth defined as ω_H - ω_L, is a small fraction of w_o with width determined by:

$$Q = \frac{\omega_o}{\omega_H - \omega_L} = \frac{f_o}{f_H - f_L} = \frac{f_o}{BW}$$

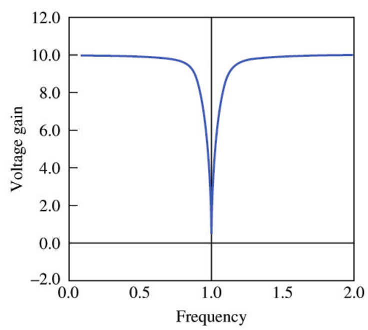
- For high Q, poles will be complex and

$$A_v(s) = A_o \frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

- Phase response is given by:

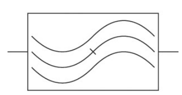
$$\angle A_v(j\omega) = \angle A_o + 90^0 - \tan^{-1}\left(\frac{1}{Q}\frac{\omega\omega_o}{\omega_o^2 - \omega^2}\right)$$

Band-Rejection Amplifier or Notch Filter



- Gain maximum at frequencies far from ω_o and exhibits a sharp null at ω_o.
- To achieve sharp null, the transfer function has a pair of zeros on the $j\omega$ axis at notch frequency ω_o , and the poles are complex.

$$A_v(s) = A_o \frac{s^2 + \omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$



Band-reject filter symbol

- Phase response is given by:

$$\angle A_v(j\omega) = \angle A_o + \angle\left(\omega_o^2 - \omega^2\right) - \tan^{-1}\left(\frac{1}{Q}\right)\left(\frac{\omega\omega_o}{\omega_o^2 - \omega^2}\right)$$

All-pass Function

- Uniform magnitude response at all frequencies.
- Can be used to tailor phase characteristics of a signal
- Transfer function is given by:

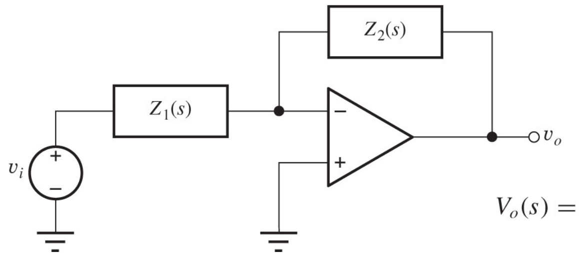
$$A_v(s) = A_o \frac{s - \omega_o}{s + \omega_o}$$

- For positive A_o,

$$|A_v(j\omega)| = A_o$$

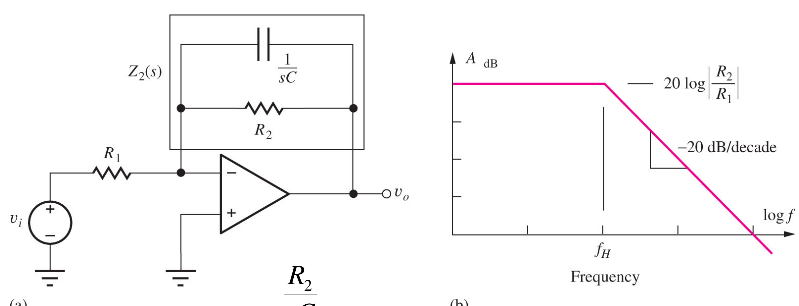
$$\angle A_v(j\omega) = -2\tan^{-1}\left(\frac{\omega}{\omega_o}\right)$$

Op Amp Building Blocks: Generalized Inverting Amplifier



$$V_o(s) = -\frac{Z_2(s)}{Z_1(s)} V_i(s)$$

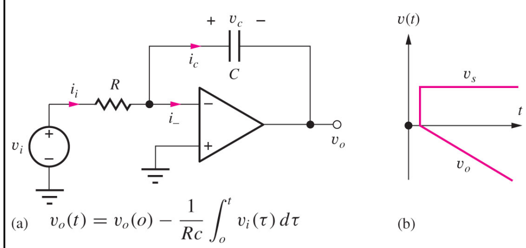
Op Amp Building Blocks: Single-pole Low-Pass Filter



$$A_V = \frac{v_o}{v_i} = -\frac{\dfrac{\dfrac{R_2}{sC}}{R_2 + \dfrac{1}{sC}}}{R_1} = -\frac{R_2}{R_1}\left(\frac{1}{1 + sR_2C}\right) = -\frac{R_2}{R_1}\left(\frac{1}{1 + \dfrac{s}{\omega_H}}\right)$$

$$\omega_H = \frac{1}{R_2C}$$

Op Amp Building Blocks: Integrator



Since $i_C = i_i$

$$\int dv_o = \int -\frac{1}{RC} v_i(\tau)\,d\tau$$

$$\therefore\ v_o(t) = -\frac{1}{RC}\int v_i(\tau)\,d\tau + v_o(0)$$

$$v_o(0) = V_C(0)$$

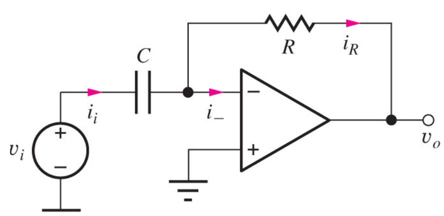
- **Feedback resistor R_2 in the inverting amplifier is replaced by capacitor *C*.**
- **The circuit uses frequency -dependent feedback.**

$$i_i = \frac{v_i}{R} \qquad i_C = -C\frac{dv_O}{dt}$$

- **Output voltage is proportional to the integral of the input**

Cal BSAC

Op Amp Building Blocks: Differentiator



$$i_R = -\frac{v_O}{R} \qquad i_i = C\frac{dv_i}{dt}$$

Since $i_R = i_i$: $\quad v_O = RC\frac{dv_i}{dt}$

- **Input resistor R_1 in the inverting amplifier is replaced by capacitor *C*.**
- **Derivative operation emphasizes high-frequency components of input signal, hence is less often used than the integrator.**

Output is proportional to the derivative of input voltage.

Cal BSAC